Use of the Chained Fisher Ideal Index to produce the Aggregated Transportation Services Index

Aggregation is the last stage in the process of producing the Transportation Service Index (TSI).

The inputs for aggregation are the separate output indexes for specific transportation modes and service types included in TSI and the value-added of each of these transportation services. Modes of transportation include air, railroad, trucking, water, transit, and pipeline. Types of transportation services are freight and passenger. Each output index is based on deseasonalized time series data for physical units such as ton-miles and passenger-miles. The value-added data covers the same time period as the output index and is measured in current dollars. Because the outputs of transportation services are measured in physical units such as ton-miles and passenger-miles, the output index used here is essentially the quantity index in the economic index literature, where output is the product of quantity multiplied by price.

The results of the Aggregation stage are three aggregate indexes that reflect the month-to-month changes and the overall trends of the output (in physical units) of the transportation sector as a whole, freight transportation, and passenger transportation, respectively.

The approach used for producing the aggregate output index is the modified Fisher ideal index method.

In the following, the specific steps for applying the Fisher ideal index method are provided:

Step 1. Definition of quantity index

The quantity index series, I_t, of a specific type of transportation service (such as railroad passenger service) puts the output of a period t, (q_t, measured in physical units such as passenger-miles), in a term relative to the output of the reference (or base) period 0, (q_0). The mathematical expression of the definition is:

(1) $$I_t = q_t / q_0$$

Step 2. Weights for aggregation

For transportation sector as a whole no single measure of output (q) is available. This is because the outputs of different types of services by the same mode (such as railroad passenger service vs. railroad freight service) or by different modes (such as railroad passenger service vs. air passenger service) are not directly comparable and additive, due to either different units or differences in quality. Therefore, we cannot calculate an aggregate output index for the transportation sector as a whole by directly implementing equation (1).

One way around this problem is to derive the aggregate output index from the output indices of the component transportation services. The logic for this approach is that since the transportation sector is made up of individual modes and types of transportation services, changes in the output of the transportation sector as a whole should be a sum of the changes in the outputs of individual modes and types of transportation services. The challenge is how to pool together the individual output indices of the component transportation services to form a single aggregate index that reflects changes in the output of the entire transportation sector. Intuitively, the aggregate index can be calculated as a weighted average of the output indices of the component transportation services:

(2) $$I_t^A = \sum I_t V_t / \sum V_t$$

Where, I_t^A is the aggregate output index for the transportation sector as a whole.

I_t is the output index of a specific type of transportation service.

V_t is the weight assigned to the specific type of transportation service. The weight can be anything that (1) reflects the importance of the service to the transportation sector as a whole; and, (2) is additive across different types of transportation services.

Value-added meets both of these criteria. It is the best indicator of the importance of a specific type of transportation service to the transportation sector as a whole and the value-added of the transportation sector is the sum of the value-added of each of the component transportation services. Therefore, in TSI, value-added the weight used to calculate the aggregate output index of transportation services. Hence, TSI is essentially a value-added weighted sum of the output indexes of the component transportation services.

Step 3. Choice of index formula

There are two issues that have to be addressed for using value-added as weights to produce the aggregate output index. One is to decide which period's value-added to use, the base period (*0*) or the target period (*t*). The other is how to avoid double counting in calculating the aggregate output index.

The first issue arises because an index number by definition always involves two periods, the base period and the target period, while weights are always associated with one specific period. For example, I_t shows the output (measured in physical unit) of year (*t*) as a ratio to the output of year (*0*). But, the value-added is always associated with a specific year, either year (*0*) or year (*t*).

The choice between historical (or base period) and current (or target period) weights is an issue discussed extensively in the literature on economic index theory. It is discussed at some length below to provide context for the discussion on the use of the Fisher ideal index formula for TSI. The Fisher ideal index contains features of both the Laspeyres index and the Paache index. Historical weights are employed by the Laspeyres index, while current weights are employed by the Paache index[1]. The widely applied Laspeyres quantity index is a specific application of the general Laspeyres index with the quantity of economic output (q) as the measurement to be indexed and prices (p) as weights:

(3) $$L(q) = \frac{\sum_{j=1}^{N} p_{j,0} q_{j,t}}{\sum_{j=1}^{N} p_{j,0} q_{j,0}}$$

The Laspeyres quantity index shows changes in quantities over time with prices held fixed at base year levels.

Laspeyres quantity index has its limitations. For example, it usually overstates the growth in real output as the current period moves further away from the base period. This occurs because quantity and price usually move in opposite directions, particularly in relative terms. That is, those commodities that increase the most in quantity tend to increase least in price over time. As a result, the use of prices from an earlier period as weights exaggerates the relative importance of the fast growing commodities as time moves on. (In contrast, a quantity index derived from Paache index usually understates the growth in real output.) The second limitation of Laspeyres quantity index is that, by fixing weights over time, it does not accommodate the effects of substitutions. This limitation is often termed as the "substitution bias" of fixed–weighted indices. Economic theory suggests that when the relative prices of commodities change over time, consumers may reach the same standard of living by substituting the commodities whose prices decreased relatively for commodities whose prices increased relatively. Depending on how prices change and how substitutions take place, the bias can be positive or negative.

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[1] The Laspeyres index is based on historical weights and is expressed in equations (a), and the Paasche index is based on current weights and is expressed in equations (b):

(a) $$L = \frac{\sum_{j=1}^{N} W_{j,0} Q_{j,t}}{\sum_{j=1}^{N} W_{j,0} Q_{j,0}}.$$ (b) $$P = \frac{\sum_{j=1}^{N} W_{j,t} Q_{j,t}}{\sum_{j=1}^{N} W_{j,t} Q_{j,0}}$$

Where, Q is the measurement to be indexed and W is the weight. $Q_{j,0}$ is the measurement for the *j*th component of a set of *N* components in the base period, and $W_{j,0}$ is the weight for the *j*th component in the base period.

The Fisher ideal index, proposed by Irving Fisher in 1922, gives good approximations to the theoretical or "exact" cost-of-living index and is relatively simple to compute and use. Mathematically, the Fisher ideal index is simply the geometric mean of the fixed-weighted Laspeyres and Paache indexes.

(4) $$F = \sqrt{\frac{\sum_{j=1}^{N} p_{j,0} q_{j,t}}{\sum_{j=1}^{N} p_{j,0} q_{j,0}} \times \frac{\sum_{j=1}^{N} p_{j,t} q_{j,t}}{\sum_{j=1}^{N} p_{j,t} q_{j,0}}}$$

Compared to fixed-weighted Laspeyres or Paache Indexes, the Fisher ideal index takes the weights of both the base period and the current period into account. By doing so, Fisher ideal index has the ability to accommodate the effects of substitutions, something the Laspeyres and Paache indexes do not do. A major advantage of Fisher ideal index over other superlative indexes, such as the Tornquvist index, is its "dual' property, i.e. a Fisher Ideal *price* index implies a Fisher Ideal *quantity* index, and vice versa. In other words, the product of a Fisher Ideal price index between two periods and a Fisher Ideal quantity index between the same two periods is equal to the total change in value (measured in current dollars) between those two periods.

These advantages lead to use of Fisher ideal index formula in building the TSI.

Step 4. Adjusting value-added to avoid double counting

The double counting issue of using value-added as weights to calculate the aggregate output index of transportation services is due to the fact that value-added itself is a product of quantity and price, i.e. $V = q \times p$. Even if there is no change in price (*p*), value-added (*V*) will increase as quantity (*q*) increases. Since quantity increases are captured in the output index (*I*), weighing the output index (*I*) by the value-added of the current period (V_t) will double count the change in quantity.

For example, let's assume that there are only two types of transportation services, railroad freight and railroad passenger. In the base period, assume that each of them had 50% share of the total value-added of the railroad industry. From the base period to the current period, there was no change in the prices of either passenger service or freight service, but the quantity of passenger service doubled while the quantity of freight service stayed the same as in the base period. Conceptually, since there was no price change, the change in quantity should equal the change in the value-added of the railroad industry-- an increase of 50% in each because of the doubling of passenger service. However, if we weigh the quantity indexes of the freight service and the passenger service, respectively, by their value-added of the current period, the change in the aggregate quantity index of the railroad industry will be larger than the change in the total value-added of the industry. In equations, this example can be expressed as:

Given: $q_t^f = q_0^f$, $p_t^f = p_0^f$, $q_t^l = 2q_0^l$, $p_t^l = p_0^l$,

and $V_0^f = q_0^f \times p_0^f = q_0^l \times p_0^l = V_0^l$,

Then: $V_t^f = q_t^f \times p_t^f = q_0^f \times p_0^f = V_0^f$, $V_t^l = q_t^l \times p_t^l = 2q_0^l \times p_0^l = 2V_0^l$

And $(V_t^f + V_t^l)/(V_0^f + V_0^l) = (V_0^f + 2V_0^l)/(V_0^f + V_0^l) = 3V_0^l / 2V_0^l = 3/2 = 1.5$

But

$$(q_t^f / q_0^f \times V_t^f + q_t^l / q_0^l \times V_t^l)/(V_t^f + V_t^l) = (1 \times V_0^f + 2 \times 2V_0^l)/(V_0^f + 2V_0^l) = 5V_0^l / 3V_0^l = 5/3 = 1.667$$

In order to get rid of the double counting, changes in weights have to be independent of changes in quantity and be a function of only changes in price. One approach to achieve this goal is to replace value-added with *adjusted value-added*, which is defined as value-added divided by the quantity index of the same period, i.e.:

(5) $$U_t = V_t / I_t = (q_t \times p_t)/((q_t / q_0) = q_0 \times p_t$$

With this adjustment, changes in value-added become a function of changes in price only. This eliminates the double counting problem but maintains the ability to capture the effects of value-added changes caused by changes in price.

$$(q_t^f / q_0^f \times U_t^f + q_t^l / q_0^l \times U_t^l)/(U_t^f + U_t^l) = (1 \times U_0^f + 2 \times U_0^l)/(U_0^f + U_0^l) = 3U_0^l / 2U_0^l = 3/2 = 1.5$$

With quantity fixed at the level of the base period (q_0), adjusted value-added is independent from changes in quantity and will change only when price changes. This makes it possible to estimate adjusted value-added using price index numbers when data on value-added are not available[2].

Step 5. Calculating aggregate output index for two adjacent periods

Applying adjusted value-added (U_t) to equation (2), we have:

[2] Since price index of a commodity is defined as: $I_t^p = p_t / p_0$, then $p_t = p_0 \times I_t^p$.
And the following equation holds: $U_t = q_0 \times p_t = q_0 \times p_0 \times I_t^p = V_0 \times I_t^p$.

(6) $$I_t^A = \sum I_t U_t / \sum U_t$$

which gives a fixed-weighted output index free of the double counting problem.

To correct the "bias" of fixed-weights, we need to the use Fisher ideal index formula to calculate the aggregated output index. However, we are calculating the aggregate output index from component output indices, rather than from component output data. The values (or levels) of the component output indices for different periods are all in relative terms to the same "base" period. Hence, a direct application of the Fisher ideal index formula will yield an aggregated output index with values (or levels) for different periods that will also be relative to the same "base' period. Though it will not cause the problem of "rewriting history"[3] when the "base" period is changed, the association of index values to a fixed "base" period may impose unnecessary inflexibility on the index. To avoid this complication, we apply Fisher ideal index formula to calculate the *change* (R_t) in the aggregate output indexes between the adjacent periods.

(7) $$R_t = \frac{I_t^A}{I_{t-1}^A} = \sqrt{\frac{\sum I_t \times U_t}{\sum I_{t-1} \times U_t} \times \frac{\sum I_t \times U_{t-1}}{\sum I_{t-1} \times U_{t-1}}}$$

Since the aggregate output index of transportation services will be a monthly index, but data on value-added of the transportation industries are available only on an annual basis, we further modify the equation to accommodate this data limitation.

(8) $$R_m = \frac{I_m^A}{I_{m-1}^A} = \sqrt{\frac{\sum I_m \times U_{y(m+6)}}{\sum I_{m-1} \times U_{y(m+6)}} \times \frac{\sum I_m \times U_{y(m-6)}}{\sum I_{m-1} \times U_{y(m-6)}}}$$

Where, the subscripts m denotes month of a year, while $y(m+6)$ and $y(m-6)$ denote, respectively, the year containing month $(m+6)$ and the year containing month $(m-6)$.

In words, equation (8) says that the monthly change in the aggregate output index of transportation services as a whole is the geometric mean of the weighted monthly changes in the output indexes of the component transportation services. The weights for each component index in month m are, respectively, the adjusted annual value-added of that component in the year containing month $(m+6)$ and the adjusted annual value-added of the same component in the year containing month $(m-6)$.

Step 6. Chaining monthly changes into a time series of indices

[3] "Rewriting history" means changing the historical growth rates from one period to the next when the base period is changed. This is a typical problem associated with the fixed-weighted indexes.

In equation (8), if $I^{A}_{m-1} = 1$, then $R_m = I^{A}_{m}$. This is because, by definition, an index measures the output of the concerned period in comparison to the output of the base period. When an index is calculated by comparing the concerned period directly to the base period, it is called "direct indexing". The procedure of direct indexing is a dual process. That is, while the output of the concerned period is indexed in terms relative to the output of the base period, the output of the base period is also indexed to 1 and only 1. Therefore, the value of a direct index is always in terms relative to 1 and equal to the change between the two periods. This property provides the mathematical underpinning for the "chain" index procedure:

Assume the output of the base period (*0*) is set to 1,

Then, $I_1 = 1 \times R_1 = R_1$

$$I_2 = I_1 \times R_2 = R_1 \times R_2$$

$$I_3 = I_2 \times R_3 = R_1 \times R_2 \times R_3$$

$$\vdots$$

$$I_t = I_{t-1} \times R_t = R_1 \times R_2 \times R_3 \times \cdots\cdots\cdots \times R_{t-1} \times R_t \tag{9}$$

In words, the index of any period to the base period can be calculated as the product of the consecutive multiplication of the changes of the adjacent periods between the base period and that period. For example, with the chain-type procedure and annual rates of changes (R_t), the index of year 1990 to year 1995 ($I_{90,95}$) can be calculated as:

$$I_{90,95} = R_{90,90} \times R_{90,91} \times R_{91,92} \times R_{92,93} \times R_{93,94} \times R_{94,95}$$

Summary

In summary, the Chained Fisher ideal index method contains two steps. In the first step, Fisher ideal index formula is used to calculate changes used in TSI in aggregate output between adjacent periods with output indexes and adjusted value-added of component transportation services as inputs. This step is also an aggregation process. Changes in aggregate output for the transportation sector as a whole calculated in this way are essentially the averages of value-added weighted changes in the outputs of the component transportation services. In the second step, changes in aggregate output between adjacent periods are chained together through consecutive multiplication to form a time series of aggregate output index for the transportation sector as a whole.

The Chained Fisher ideal index method recognizes the need in estimating changes in the (quantity) output of transportation services as a whole to use weights that are appropriate for the specific periods being measured. The Chained Fisher ideal index method has three important advantages over Fixed-Weighted Indexes. First, while it allows aggregation of different transportation services into one measure through weighting, it also captures the effects of changes in the relative importance of different transportation services over time. Second, it minimizes substitution bias and, at the same time, provides a more accurate description of the cyclical fluctuations in the output of transportation services as a whole. Third, it eliminates the inconvenience and confusion associated with Fixed-Weighted Indexes of updating the weights and base periods, and thus avoiding rewriting economic history, as base periods move further and further into the past and become more and more irrelevant to the current period.